FROM
KANT to EINSTEIN

BY

HERVEY DE MONTMORENCY

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Contents.

(Research)

From Kant to Einstein.

I.

INTRODUCTORY REMARKS.

KANT, in his CRITIQUE OF PURE REASON, declares: "time does not change, but phenomena change in time;" also, "everywhere space has three dimensions and cannot in any way have more," and again, "there is only one time and there is only one space." Thus Kant's fundamental idea is this: within the scope of our knowledge, time and space are absolute.

On the other hand, the basis of Einstein's theory is the relativity of time and space.

It is the object of this inquiry to ascertain if it be possible to reconcile these conflicting views; and perhaps, owing to their high prestige, Kant's doctrines may serve as a canon for judging Einstein's propositions.

It is, therefore, of the utmost importance at the beginning of our task to point out the necessity for understanding Kant's views on time and space. These are set out very clearly in the TRANSCENDENTAL AESTHETIC; but it is also essential to study the chapter in the CRITIQUE OF PURE REASON, on the schematism of the categories, which shows how time is the universal form of all internal sense, and how all conceptions and judgments must be brought into relation with time, because it is in that connection that Kant may be found to differ from philosophers of the school of Bergson. This will be manifest when Einstein's propositions are examined.

The keystone of the arch supporting Kant's metaphysics is his explanation, in the PROLEGOMENA, of the nature of mathematical judgments, and their nature, too, may have a bearing upon some of Einstein's demonstrations.

II.

THE SPECIAL THEORY OF RELATIVITY.

THE task which Einstein set himself was that of reconciling the classical principle of relativity with the constancy of the velocity of light: two maxims which appear to be incompatible.

The principle of relativity postulates that, if A move towards B with a speed of 40 miles an hour and B towards A at 20 miles an hour, the relative velocity of A with respect to B—or of B with respect to A, because relative velocity is reciprocal—will be 60 miles an hour. The constancy of the velocity of light seems to contradict this principle of relativity, inasmuch as the velocity of light is always 300,000 kilometres a second to the terrestrial observer, whether he be at rest or move rapidly towards or from the source of light.

Einstein's solution of the problem is his Special, or Restricted, Theory of Relativity, which is simply this: Time and Space are not absolute but relative. Thus, if an event occur on a body which is moving with high velocity with respect to an observer who is stationary, then Time and Space, which are the conditions under which that event and its concomitant phenomena are perceived by the observer, will be distorted. Space will be contracted (shrunk), Time will be distended (stretched), and simultaneity will be broken. All space-measurements, therefore, must appear shorter to him and time-intervals (measured by clocks) slower; they will be seen as it were, in a sort of perspective. Moreover, the faster the relative velocity, the greater will be the distortion, until the speed of light be attained, when so considerable will become the contraction of space and the distension of time that nothing can be perceivable; the velocity of light being, in the jargon of mathematicians, the limiting velocity.

For instance, if, from a body A, which is moving at the rate of 200,000 kilometres a second with respect to an observer B, another body D be projected forward at 200,000 kilometres a second in relation to A, then the velocity of D in relation to B will not be 400,000 kilometres, as might be supposed, but, by means of a Lorentz-Einstein equation of transformation, to which reference will be made, it can be calculated that it will be only 276,923 kilometres, because what appear to be kilometres to A will be shrunken to B.

Again, if, in the above instance, A have a velocity of 299,999 kilometres a second and D be projected forward, as before, with a velocity of 200,000 kilometres a second, then, owing to the greater velocity of A from which D has been launched, the kilometres will appear still further shrunken to B; they will have shrunk so much, in fact, that the increment will only appear to be one kilometre a second to B, and the total resulting velocity will be only 300,000 kilometres a second to him; that is the velocity of light which can never be exceeded.

In the brief explanation which has been given, for the sake of conciseness, we have alluded to the shrinkage of spatial measurements only, but the velocity of an object is the rate of its motion through a distance of space in an unit of time, so that the distension of time is a factor of equal importance in solving the puzzle of the constant velocity of light. In all the Lorentz-Einstein equations it will be seen that velocity and so, implicitly, time, is an implicit function of the mathematical results which are determined.

These equations are of unknown quantities referred to four co-ordinates: the three spatial co-ordinates and the time co-ordinate.[1]

All events, therefore, may be described as taking place in a four-dimensional space-time continuum.

This is Einstein's explanation of the constant velocity of light, a phenomenon which had puzzled scientists for years, because it has been experimentally proven that, when light is projected from a body rapidly approaching the observer it

[1] See Appendix.

appears to him to be travelling at the same velocity as when projected from a body which is stationary with respect to the observer.

It is most important to bear in mind that the constant velocity of light is the *raison d'être* of Einstein's Special Theory. Indeed, it is vital to it.

In his book, THE THEORY OF RELATIVITY, Einstein writes thus: "By means of considerations based on observations of double stars, the Dutch astronomer, De Sitter, was able to show that the velocity of propagation of light cannot depend on the velocity of motion of the body emitting the light."

Again he writes: "The epoch-making theoretical investigations of H. A. Lorentz . . . lead conclusively to a theory of electro-magnetic phenomena of which the law of the constancy of the velocity of light is a necessary consequence."

A six-foot man seen at a distance appears reduced in height. He is really still a tall man, but distance makes him seem small, because he is seen in perspective. That which makes all moving objects appear contracted to an observer who is at rest, according to Einstein's Special Theory, is something analogous to perspective; whereas distance makes the six-foot man appear to be a dwarf, speed (in relation to the observer) makes the measured distance seem foreshortened; and it makes the clock which is moving appear to go slower.

Although an humble mortal might be unable to illustrate or explain perspective, while a Velasquez might express it by colour, or a Michael Angelo in line, nevertheless it is in the nature of all human minds to apprehend perspective.

It has been erroneously asserted that, according to Einstein, if Jack were a passenger on a cannon-ball fired at high velocity—a velocity comparable to that of light—to some remote star, and if, on arrival at the star, he were immediately shot back to the Earth, it might come about that, although Jack would only be two years older after his journey, Jill, who from the earth had watched his flight to the star and back, would be two hundred years older when she greeted Jack on his return.

Now this is certainly not correct according to Einstein. His interpretation of the fantasy might be as follows:—

To Jill, Jack's flight would appear to be of two years' duration *as recorded by Jack's watch ;* but Jack would really have lived two hundred years during the journey, just the same as Jill. On the other hand, to Jack, riding on the cannon-ball, it would appear that the earth had dropped away from him and returned to him in two years' time *measured by Jill's watch ;* but both Jack and Jill would be two hundred years older.

The Time lived is the real time, but the time of the flying cannon-ball, seen in perspective[1] by Jill, makes Jack's watch seem to go slowly to her—to have only marked two years in two centuries.

Let us put this fantasy in another way. Suppose Jack and Jill, having synchronised watches in their hands, to be riding together on the cannon-ball; let Jack step off the flying cannon-ball on to the earth. It will then appear to him that Jill's watch is going much slower than his own; on the other hand, Jill will think that Jack's watch is going much slower than hers.

Another fictitious legend which has cropped up in connection with Einstein's Theory is that, owing to the dislocation of simultaneities, an observer on this earth might be able to witness future events on another planet moving rapidly with respect to himself. This is not so.

The breaking of simultaneities is analagous to the double vision we sometimes obtain of one object owing to astigmatism, or faulty focussing in our eyesight. It does not mean that one event is projected into the future and another recoils into the past.

The difficulty of defining simultaneity has engaged the ingenuity of many philosophers, and Bergson has distinguished between natural and intuitive simultaneity. It does not follow, because two or more events are intuitively

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[1] The use of the word perspective may give the erroneous impression that this contraction of space is an optical effect; but this cannot be so, as the distension of time clearly shows that the delusion is more general in character; it is a misleading of the perceptive faculty of the mind.

simultaneous to an observer, that they are naturally simultaneous. Nevertheless whatever system of synchronised clocks be employed, the difficulty of transmitting any information recorded by the clocks across a distance of space is a difficulty which is a function of the intuitive simultaneity which we believe can be defined as the presentation to the mind of two or more events in a single, instantaneous perception. It follows that the intuitive simultaneity of two events depends on the point of view of the observer of those events. If, for instance, two flashes at A and B appear to be simultaneous to an observer posted exactly mid-way between them, they will not appear simultaneous to an observer who is close to A and remote from B, because the light ray from the flash at B will take longer to reach him than that which comes from A.

Similarly an observer in motion with respect to the system containing A. and B where the flashes occur, sees all space-measurements in that system foreshortened and time, the condition under which he perceives all phenomena, is lengthened according to Einstein's theory. Nevertheless, the interval apparent between two events which appeared simultaneous from another point of view, cannot contain any fresh event.

Einstein's Special Theory explains the relativity of all systems moving uniformly and rectilinearly with respect to one another. That is to say, it is concerned with movements of translation only—movements which are altogether exclusive of rotation. Einstein's General Theory has no such limitations.

In the example which was given above, when the velocity of A and the velocity of D's projection from A are known and the velocity of light has been taken into consideration, the resultant velocity of D with respect to B, the observer, can be determined by means of Lorentz-Einstein equations of transformation and these preserve their form for all systems in Einstein's Special Theory, though, as it will be seen, they are not adequate for the solution of problems in his General Theory in which he has to resort to Gaussian equations.

In these Lorentz equations, it will be observed that the velocity of light is always the unit, or standard, by which other velocities are measured, and as velocity implies time-interval as well as space-interval, four dimensions come into consideration: time and the three ordinary spatial dimensions hence the space-time continuum in which every event occurs.

Before the Theory of Relativity had been propounded, time played an independent part in the consideration of events, and their concomitant phenomena; but Minkowski explained that the world of physical phenomena is four dimensional; because, to fully explain events, they must be referred to four co-ordinates: the three spatial co-ordinates and the time co-ordinate. Einstein writes thus:—

"The four-dimensional mode of consideration of Minkowski's 'world' is natural in the theory of relativity, since according to this theory time is robbed of its independence." This is shown by the fourth equation of the Lorentz transformation.[1]

Einstein states: "General laws of nature are co-variant with respect to Lorentz transformations," and, from this, he deduces some modifications of the classical laws of the conservation of mass and energy.

This, then, is the Special Theory of Relativity which has caused so much sensation.

The point to be borne in mind is that it is not merely the yard-measures which are shrunken (physically), that, indeed, was the explanation advanced by FitzGerald and Lorentz in the first instance to explain the constant velocity of light; but, in the Einstein Theory, Time and Space themselves—the very *a priori* forms of our sense—are affected, so that everything which we perceive must be proportionately affected; yard-measures must appear contracted and clocks to go slowly.

It is just as impossible for us not to perceive Space and Time-intervals distorted, when translated swiftly past us, as it would be impossible to see a six-foot man appearing fully two yards high, when seen half a mile distant.

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[1] See Appendix.

It is worth remembering, too, that contraction takes place in the line of movement only; thus a circle, moving across the line of sight at a high velocity with respect to an observer who is at rest, will appear to him as an ellipse, the minor axis of which is in the direction of advance, its major axis remaining of the same length as the circle's diameter.

The reason why the classical principle of relativity has been accepted as true since the days of Galiléo is that the velocities, with which we are acquainted in everyday life, are so small compared with that of light, that the errors, which arise from the mere algebraical addition of the velocities of the bodies concerned in order to find their relative velocity, cannot be detected.

III.

THE GENERAL THEORY OF RELATIVITY.

IN the GENERAL THEORY OF RELATIVITY, Einstein deals with the more complex questions of events which occur in gravitational fields and where relative movements of rotation have to be considered; he tackles the great problems of the motions of the heavenly bodies and throws a new light upon the nature of gravitation.

He points out that gravitation cannot be a force, because the heaviest and the lightest objects fall to earth (*in vacuo*) with equal acceleration under its influence, whereas an engine of given horse-power can draw a light carriage more easily and rapidly than a heavy carriage.

Just as a magnet calls into being, in the surrounding space, a certain physical state which is known as a magnetic field, so, according to Einstein, every heavenly body envelops itself in a gravitational field which is a sort of vortex of curved space. As Professor Eddington says: "Wherever matter exists space-time has a curvature." This gravitational field diminishes in intensity with the distance from the centre of the body; moreover, the greater its mass, the intenser and more far-reaching the field.

Just as, in the Special, or Restricted, Theory, space and time are affected by the velocity of the observer in relation to the system being studied, so, in the General Theory of Relativity, distance and velocity are affected by the potential of the gravitational field: the intenser the field, the greater the contraction. Thus, in measuring the distance from a point, situated in a gravitational field, to another point in an intenser zone of that field, the units of measurement become more and more contracted, the nearer they approach the centre of the field; the distance so progressively contracting being a tensor.

In contrast, however, to electric and magnetic fields, the gravitational field gives evidence of a remarkable property, as bodies, moving under its sole influence, receive an acceleration which is independent of their material or physical state. Thus a ton of lead and a feather fall (*in vacuo*) in exactly the same manner in a gravitational field.

Nothing can pass through a gravitational field in a straight line, but everything must follow a curve, because the ambient space is curved and in curved space there can be no straight lines, just as no straight line can be drawn upon the surface of a sphere or of an egg (an ellipsoid).

The shortest distance between two points upon a plane—a two-dimensional surface—is a straight line, the shortest distance between two points upon the surface of a sphere is an arc of a great circle of that sphere, and the shortest distance between two points in curved space is a geodesic line, or what Einstein terms an "interval." It is important, however, to remember that this "interval" is not merely the spatial distance between the two points, but it depends as well on the velocity (implicitly the time) of an object passing from one of the points to the other; in the language of mathematicians, the "interval" is the explicit function of both time and space.

A ray of light always follows the shortest path. This was a postulate to the classical geometers and physicists who confidently believed that the shortest distance between two points must always be a straight line; but, in passing through a gravitational field, a ray of light must follow a geodesic line and, in curved space, a geodesic line (the shortest path) must be a curve.

The amount of curvature of a ray of light from a star grazing the sun and passing through the sun's gravitational field has been calculated by Einstein, and the verification of his results by many experiments carried out during solar eclipses has been one of the most triumphant vindications of his theory.

Not only must a ray of light always follow a geodesic line through a gravitational field, but so must all bodies, whatever may be their material or physical state. In entering a gravitational field, they are caught in the vortex and must swing round, like the planets and satellites, in orbits which are geodesic lines, each pursuing its shortest path through the curved space, the path depending upon the intensity of the field and the initial velocity.

Howbeit, if space, which is three-dimensional, be curved, the question of a fourth dimension demands attention.

If we assume a straight line to be one-dimensional, it follows that a curved line must have a second dimension into which it may bend, that is to say, a plane which has width as well as length.

Now let us imagine a sheet of paper (lying flat) to be a two-dimensional body, or plane, the thickness of the paper, its third dimension, being negligible. Before this sheet of paper can be warped or crumpled, it must have a third dimension into which it can expand, so, by analogy, we can conclude that, if a three-dimensional body be bulged or curved, there must be a fourth dimension into which it can extend itself.

Professor Eddington, carrying this argument further says: "Our four-dimensional space-time may be regarded as a closed surface in a five-dimensional continuum; it will then be finite although unbounded, just as the surface of a sphere is finite but unbounded."

Einstein introduces time as a fourth dimension in his Special, or Restricted, Theory of Relativity, thus, apart from the consideration of gravitational fields and before envisaging the problem of the curved space of a gravitational field to which we are introduced in the General Theory of Relativity, we have to recognise that all events occur in a four-dimensional space-time continuum, in what Minkowski terms the "world;" so that it is this "world" with its concomitant phenomena and events—or at least the space of the space-time continuum—which is embedded in another dimension.

Einstein has developed his General Theory from Minkowski's "world," as he writes: "without Minkowski's idea the General Theory of Relativity would perhaps have got no further than its long clothes."

That which, in the old-fashioned domain of knowledge, where time was independent of space, would have been termed an event is called, in the space-time-continuum, a world-point.

In Einstein's words: "From a happening in three-dimensional space, physics becomes, as it were, an existence in Minkowski's 'world'"

The shortest path between two world-points, or, in other words, the distance between two events measured as to both time and space is the "interval" to which reference has been made.

Einstein has calculated that, in a given gravitational field of a given intensity, the "interval" for two world-points is constant.

A straight line, an arc of a great circle and an "interval," are all geodesic lines, or shortest (most convenient) paths, according to whether they be traced in empty space—where no disturbing masses whatever are present—on the surface of a sphere, or in the curved continuum; in the jargon of the metaphysician, they are only different modes of a geodesic line. Thus one of the great stars, for instance, alone in empty space, remote from any of the perturbations caused by other masses, if such an imaginary condition were possible, might be expected to follow a straight line, but, in a gravitational field, a planet encountering curved space, has to follow its geodesic line—its orbit—through the sun's gravitational field.

It may be asked why, if, in a given gravitational field all intervals be constant, does not the orbit of Mercury coincide with that path of a light-ray which passes near the planet and through the sun's gravitational field, as both should follow geodesic lines?

The answer is this: both do follow geodesic lines, but Mercury's geodesic line is very different from that of a ray of light, because the velocity of Mercury is very small compared with that of a light-ray. In calculating the geodesic line traced by the planet through the sun's gravitational field, Einstein has to take into consideration Mercury's velocity; both the distance covered and the time consumed in covering that distance come into the problem; both space and time are factors; the spatial distance between, say, perihelion and apohelion, as well as the time taken for the journey must be known; when these two quantities have been determined, then the geodesic line, or the "interval," between the two world-points, one of which is Mercury's passage through perihelion and the other the passage through apohelion, can be calculated.[1]

Thus it can be understood that Mercury's path must differ enormously from that of a ray of light, though both must follow geodesic lines, and though the "intervals" of those world-points which are in the same intensity of the same gravitational field must be the same. Generally speaking, the greater the gravitational intensity, the greater the curvature of the geodesic line; the greater the velocity, the less the curvature; as with a bullet, the higher the velocity, the flatter the trajectory.

Einstein writes thus: "If we confine the application of the theory to the cases where the gravitational fields can be regarded as weak in intensity and in which all masses move, with respect to the co-ordinate system, with velocities small compared with that of light, we then find a close approximation to the Theory of Newton. Nevertheless, as the meticulous accuracy of calculations develops, deviations from it make their appearance.

According to the Newtonian theory, when every allowance has been made for the perturbations due to the gravitational influence of other planets (and for aberration), we ought to obtain for the orbit of each planet an ellipse which is stationary with respect to the fixed stars.

This deduction, which can be tested with considerable accuracy has been confirmed for all the planets save

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[1] The distance between two world-points may be expressed thus in the form of a Cartesian equation:—

$$S^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - (t_2 - t_1)^2$$

where x, y and z are the usual spatial co-ordinates, and t is the time-co-ordinate; but, as a concession to a metaphysical doubter, who regards time, in this connection, as a mere mathematical entity, we can put $t = \tau(\sqrt{-1})$: an imaginary quantity.

Mercury"—which, be it remembered, is the nearest to the sun of all the solar system and is, therefore, in the intensest part of the sun's gravitational field—" It has been found that the ellipse of Mercury's orbit is not stationary, but that it rotates exceedingly slowly in the sense of the orbital motion and that it advances 43 seconds of arc in a century.

Now the *General Theory of Relativity* teaches that the ellipse of every planet must necessarily advance, as does that of Mercury; but that, for all the remoter planets, the rotation is too small to be detected, but that, in the case of Mercury, it must amount to the very 43 seconds of arc in a century which had been determined by observations."

The path of a planet, according to Einstein, is very nearly an ellipse, but it does not quite close up; there is a gap, so, in the following revolution of the planet, the orbit has to advance slightly in the same direction as that in which the planet is moving.

Draw an ellipse on a piece of paper, cut it out and slit it along a radius-vector from the circumference to a focus; then place this focus on the apex of a cone and endeavour to wrap the paper around the cone; you will find that there will be an overlapping.

This is Professor Eddington's experiment for giving a rough idea of the advance of Mercury's perihelion.

In the above experiment, the apex of the cone represents the sun, the surface of the cone is Einstein's curved space, the edge of the paper represents the orbit of Mercury (or any planet) and the overlapping gives an idea of how the advance of Mercury's perihelion is caused; it shows the second revolution of the planet closing the gap in the previous revolution.

Einstein contends that "a gravitational field is exactly equivalent to a field of force introduced by a transformation of the co-ordinates of reference, so that by no possible experiment can we distinguish between them." The effect of this equivalence is that no mere sensation can be a guide in any enquiry into the nature of gravitation. The sensation of being drawn up in a lift is simply the same sensation which would be experienced if there were suddenly an increase of gravity; if there were no gravity, and if we were being drawn up in a lift, we should have the same sensation as if we were suspended at rest in a gravitational field. Briefly, weight and inertia are the same thing.

Newton, on the other hand, considered weight and inertia to be two entirely different things, but fortuitously of equal value.

Einstein denies that gravity is a force, as it is regarded in Newton's theory of gravitation. He considers gravity to be a physical condition brought into being by every heavenly body and with which that body surrounds itself. He contends that the intensity of this condition, which he terms the gravitational field, varies with the mass of the body, but that its mass, varying with its energy, and so implicitly with its velocity, is not constant, but increases and decreases with its velocity. Einstein agrees approximately with Newton's law prescribing that gravity varies inversely as the square of the distance, if it be borne in mind that the intenser the gravitational field, the shorter are the units of measurement, the distances shrinking as they approach the sun, and stretching as they recede from it. We may obtain a rough idea of this system of tensors if we suppose that, in measuring distances between the sun and Mercury, we use a chain divided into feet; that, in measuring between Mercury and the Earth, the feet—unbeknown to us—become yards, and between the Earth and Uranus, the yards are still further stretched into fathoms.

Newton accounts for the orbit of a planet thus:—its path is the resultant of two forces: (1) The force from which its initial velocity was derived; (2) The force of gravity pulling it towards the sun.

Einstein describes the orbit of a planet as the geodesic line, or the shortest (most convenient) path adopted by the planet in which to travel through the curved space of the sun's gravitational field, the velocity of the planet not being sufficient to carry it through the vortex of this field, its path curves round into itself in an ellipse.

IV.

NOTES AND COMMENTS

MANY of the greatest mathematicians have been metaphysicians—Des Cartes and Leibnitz, for example—but Albert Einstein, as far as it is possible to judge from his writings, is simply a mathematician; indeed, it might be more accurate to describe him as a physicist who has displayed the utmost skill and ingenuity in applying the mathematical discoveries of Gauss, Minkowski, Riemann and Poincaré to the problems of physics.

In some of his demonstrations he certainly makes it clear that he neither subscribes to, nor understands Kant's doctrine regarding the nature of mathematical judgments. Thus Einstein contends that, if a measuring rod be applied to the periphery of a rapidly rotating circular disc and then to its diameter, it will be found that the circumference of the disc is not 3.1415 etc. times the length of its diameter and, from this, he concludes that the value of π is not always the same under varying conditions of observation.

Of all Kant's arguments none, perhaps, is so convincing as that in which he contends that mathematical judgments are synthetic judgments *a priori*, that is, before experience; but Einstein's experiments with measuring rods and discs in rotation yield results which are obviously *a posteriori*, that is, after experience. His judgment, therefore, in the case of the rotating disc, is not mathematical at all.

The value of π was not discovered through experimental methods, but from the geometrical figure known as the circle which has no objective reality, but which is a schema created in the human mind. It has certain properties which can be determined by mathematicians *a priori*, not by empirical judgments *a posteriori*.

Kant says: "Geometry is based on the successive synthesis of productive imagination in the generation of figures."

It is true that he also says: "Whatever pure mathematics proves to be true of space and time must be true of all external objects." Nevertheless some pure mathematicians, Riemann, Christoffel Ricci, never dreamt of applying their theory of tensors physically. Besides, it must be impossible to prove that Einstein's spinning disc has a periphery which is a true geometrical circle.

Again, Einstein writes: "I cannot conceive the motion of a body in space; a body must move in relation to some standard."

But Einstein also writes: "An isolated body, remote from all perturbing influences, must move in a straight line." So that he can, and does, actually conceive an isolated body moving through space.

If Einstein were a metaphysician, he would choose his language with more care. He might say: "I cannot perceive any motion except relative motion: that is, motion with respect to some standard; but, in my imagination, I can conceive the idea of an isolated body drifting through space."

Many of Einstein's arguments are open to criticism and contain internal evidence of carelessness. Thus, with reference to simultaneity, he declares:—"Two flashes which occur at A and B and appear simultaneous to an observer M, who is standing still midway between A and B, will not appear simultaneous to him if, instead of standing still, he be moving rapidly, let us say, towards B, because M is hastening towards the beam of light coming from B, whereas he is hurrying on ahead of the beam of light coming from A; hence he will see the beam of light coming from B earlier than that which has been emitted from A."

But this can only be the case if the relative velocity of the beam from B be greater than the relative velocity of the beam from A, because M, although in motion, is, by hypothesis, still equally distant from A and B; so that the velocity of the beam of light from B must appear to M to be greater than the velocity of light, which, according to Einstein himself, is impossible; the constancy of the velocity of light being, as we have shown by quotations from his writings, the fundamental basis of Einstein's Special theory.

The simultaneity is broken, not because M is hurrying ahead of the ray of light coming from A and towards that which is approaching from B, but because M, being in motion in relation to the points A and B, space, the one *a priori* form of M's perception, is contracted, while time, the other *a priori* form, is distended for the perception of all phenomena occurring at A and B.

The fourth Lorentz equation of transformation[1] shows that, even when the time-difference vanishes between two events occuring in a system K, the time-difference does not vanish to an observer moving with respect to K.

Again, he contends that, when a measuring rod is applied to the periphery of a rapidly rotating disc, it will appear shortened, in accordance with his Special Theory of Relativity; but, when the same measuring rod is applied to the diameter of the same disc at right angles to the direction of motion, it will not be shortened. It is from this experiment that he deduces his contention that the value of π is not 3.14159 etc. in a revolving circle.

But, if the rod be shortened, so, too, is each length of the periphery as it is measured, according to Einstein's Special Theory, both must shrink together, the measure itself and the span to be measured, so that, in comparing the shrunken rod with a shrunken arc of the spinning periphery, no shrinkage will be detected.

Professor Eddington deals with this argument concerning the value of π thus, If a mass be supposed to be at the centre of a circle, the gravitational field created by this mass will cause any radius to be contracted; it will become a tensor, so that the ratio of the circle to its diameter will be greater than 3.14159 etc.

Our answer to this is, that the diameter of a true geometrical circle in plane geometry is a schema created in the mind; it is not a string, nor a muscle, nor an elastic band, nor any

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[1] See Appendix.

physical thing capable of being stretched or contracted. It is a line (distance), not a tensor!

There is nothing, however, in this argument to contest the justification for the solution of mathematical problems based on a hypothesis that the radii-vectores of a curve traced between a focus of that curve and its circumference must be tensors; only the curve will not be a true circle.

It is, of course, quite obvious that Einstein has never carried out in practice any of his experiments with measuring rods or spinning discs. They are merely illustrations. His railway-trains travelling *in vacuo* along embankments, as devoid of atmosphere as any pre-Raphaelite landscape, exist only in his imagination.

Railway-trains and discs are employed by Einstein to illustrate his arguments, as the apples and nuts are used in a kindergarten to teach addition and subtraction to children.

V.

HOW MATHEMATICS IN FOUR DIMENSIONS IS POSSIBLE.

It may not be out of place here to give some explanation of how mathematical problems in four dimensions have become possible.

On a plane, Des Cartes drew two straight lines at an angle, so that they cut one another at a point known as the origin. These two straight lines he called the "abscissa" and the "ordinate," or the co-ordinates x and y.

By noting its distances from these two lines, he was always able to determinate the exact position of any point in the plane.

As a curve is only a succession of an infinite number of contiguous points, he was able, by successive measurements to these points from the co-ordinates, to arrive at a mathematical expression—an equation to the curve—describing the curve as referred to the co-ordinates, x and y; also, vice versa, given an equation, he was able to plot its curve by applying different solutions as measurements from the co-ordinates on the plane: the curve so plotted is called the graph, or picture, of the equation.

These Cartesian equations, as they are termed, are always in two unknown quantities and of the second degree; that is to say, the two unknown quantities are never of a higher power than the second, thus: x^2 and y^2.

Now, by drawing a third co-ordinate, z, we can extend this principle so as to form an equation to a spiral; not, of course, on a plane, but in a volume of space. The equation, in this case, will be in three unknown quantities and of the third degree, thus: x^3, y^3 and z^3. As space, however, has only three dimensions, it is not possible to draw more than three straight lines through the origin at right angles to one another, so here we must come to a stop. We can do no more than plot a curve on a plane, from its particular equation, or a spiral in a spherical space.

Nevertheless, it is possible to write down an equation in four unknown quantities and of the fourth degree, and we are justified in assuming that this may be an equation to something analogous to a spiral referred to four co-ordinates. Thus, if we could plot the graph, or picture, of this equation, we would find that it was a line bending and twisting in four dimensions.

The human mind, however, is quite incapable of visualising four dimensions, because the condition of all our external sense is space which is three-dimensional. It is even doubtful whether we can visualise an object of one or two dimensions, because however thin a plane may be, it must have some thickness (a third dimension) to exist at all in our perception, so that our perceptions are cribbed, cabined and confined within three dimensions. Nevertheless we can certainly form some sort of conception, in our imagination, of a straight line or of a two-dimensional plane. We can form schemata of them, perhaps even mental images, notwithstanding the limitations of our perceptions; but we can most certainly not form any sort of image, schema, conception, or idea of a four-dimensional object, even in our very wildest flights of imagination. Nevertheless, it is possible to solve mathematical problems in four dimensions.

Let us take an example. Suppose we have four unknown quantities; if we can determine any sort of relations between them, we can write down a mathematical formula covering those relations, then, if we operate three successive integrations of the formula, by aid of the Integral Calculus, we will obtain an equation of the fourth degree in four unknown quantities; that is, an equation to some line (analogous to a spiral), referred to four co-ordinates; otherwise a line twisting and curving into four dimensions.

Now, Gauss solved problems in more than three dimensions by the aid of his numerous and curvilinear co-ordinates. The Gaussian system is a logical generalisation and extension of the Cartesian system; an infinite number of curves, designated by the letter *u*, taking the place of the abscissa in the Cartesian system, while other transversal curves, designated by the letter *v*, take the place of the ordinate.

Gauss invented this method for the mathematical investigation of continua in general. To every point of a continuum are assigned as many Gaussian co-ordinates as the continuum has dimensions.

Einstein solves his problems by the aid of a system of Gaussian co-ordinates.

VI.

EINSTEIN'S DIFFICULTIES: THE LACK OF RIGIID STANDARDS OF REFERENCE: TIME AS A FOURTH DIMENSION.

WHEREAS, in his Special Theory, Einstein tackles problems in what he calls Galiléian domains, which are those in which no gravitational fields exist, in his General Theory he cannot escape from the necessity of considering the effect of rotary movements.

In Einstein's own words: "There are no such things as rigid bodies with Euclidian properties in gravitational fields; thus the fictitious rigid body of reference is of no avail in the general theory of relativity.

The motion of clocks is also influenced by gravitational fields and, in such a way that a physical definition of time, which is made directly with the aid of clocks, has by no means the same degree of plausibility as in the special theory of relativity. For this reason, non-rigid reference-bodies are used which may appropriately be termed reference-mollusks."

Heretofore, every astronomer and physicist, in making his calculations, has relied on some fixed standard or system of co-ordinates to which he could refer his measurements, has laid to his soul the flattering unction that something in the universe must be absolute; but Einstein has removed this illusion. In the domain of science, there is no rock on which to build a faith; everything is always shifting in position and changing in value, so that Einstein's bodies of reference are neither solid, fixed, nor absolute. That is what he means when he calls them mollusks.

If we imagine a frame-work of metal rods to be so arranged as to represent a fixed system of Cartesian co-ordinates, and if we suppose considerable heat to be applied to certain portions of it, the metal rods, under the influence of the varying temperatures, will begin to expand, and the whole frame-work will be warped; such a warped and twisting frame-work will give an idea of Einstein's reference-mollusk.

The Lorentz-Einstein equations of transformation which preserve a resemblance to those of Des Cartes are no longer adequate for the general theory of relativity, and Einstein describes his reference-mollusk as being "the main equivalent to a Gaussian four-dimensional co-ordinate system chosen arbitrarily."

The four-dimensional system in question is Minkowski's Space-Time-Continuum, or "world."

The idea of time as a dimension may be developed in the following manner. Let us suppose the corner of a room to be the origin and the three lines formed by the intersection of the floor and two walls, to be the three spatial co-ordinates. Then it is possible to determine the exact position of a lamp suspended in the room by taking measurements from it to these three co-ordinates. But suppose we wish to record when the lamp was lit or when extinguished, then we have to note the exact time at which either of these events occur by reference to a clock, so that a time-co-ordinate enters into the consideration of all the events relating to the lamp.

Navigators reckon their longitude—which is a co-ordinate vital to navigation—by reference to a chronometer; and so the idea crops up of a time-co-ordinate in similar problems, such, for instance, as the tracing of a monthly weather-chart by a barometer fitted with a revolving drum.

Velocity being dependent upon both space and time, and being an element in calculating the "interval" between two world-points, as has been already explained, the time-co-ordinate is essential to Einstein's General Theory. Indeed, so wedded is he to the notion of the time-co-ordinate that he writes: "That which gives the mollusk a certain comprehensibleness as compared with the Gaussian co-ordinate system is the (really unjustfied) formal retention of the separate existence of the space-co-ordinates as opposed to the time-co-ordinate."

It seems to us that Minkowski and all who have succumbed to his fascinations have simply conceived a four-dimensional continuum and have arbitrarily called its fourth dimension time.

Einstein has been lured on to this dangerous ground by the Lorentz equations. In all Lorentz-Einstein equations of transformation, the velocity of light is the important factor; it is the unit, or standard, by means of which other velocities are measured.

Now velocity involves space and time. It is the spatial distance covered in an unit of time; in the language of mathematicians, velocity is the explicit function of space and time,[1] each of which is an implicit function of velocity.

So, that, implicitly, time has crept into every one of the Lorentz equations.

Lorentz uses a symbol for the speed of light—let us say C,—and another symbol—let us say V—for the velocity of the body being studied. These are quite justifiable steps in these mathematical operations, nor can any error arise, because, in the Lorentz equations, it is the ratio between V and C (V measured by C) which affects the result. Indeed, such is the case in all Einstein's propositions. The amount of the annual advance of a planet's perihelion, for instance, is $3\frac{v^2}{c^2}$ of its revolution.

But it is grotesque to suggest that anything in the nature of real time—the time during which events occur, the time with which we associate the consciousness and memory of an inner flux of sensations—is implied by these symbols.

C and V and their values are mere mathematical entities which may signify the quantity, but can never express the quality of time.

C, to Einstein, for instance, in the Earth's gravitational field, is 300,000 when working out his problems; 300,000 kilometres a second. But this 300,000 is a mere number; 300,000 kilometres is mere distance. Where does time,

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[1] Or it is the derived function of distance (space) with respect to time.

considered psychologically, or metaphysically, influence the problems?

How can time, in which we suffer, rejoice, hate, or love, and grow old, be a dimension of space? How can time, in which a memory lingers, be a Gaussian co-ordinate?

The idea of time being subordinate to space must be repugnant to any Kantian. "Space," says Kant, "is the image of external magnitudes, Time the image of all, whether external or internal."

The importance of time in each and every process of thought, is essential to Kant's philosophy. He explains this at great length in the schematism of the categories; every perception, judgment, conception, or idea in the whole complex process of reason is woven into time.

Kant finds that, through one point, only three straight lines can be drawn at right angles to one another, and so he concludes that space can only have three dimensions.

Now let us consider how Kant explains the mental process of constructing these lines. "In imagination,"—he declares —"we form an image of each of these lines by supposing a succession of points, one added after the other chronologically until we have created the line." That is to say, we have a series of contiguous points presented to our minds in time; they are sense-data threaded on time, the series in time being of the essence of the conception.

Thus every step in determining the dimensions of space, according to Kant, involves the use of time. To satisfy Minkowski, therefore, the greater must become the lesser; time, which embraces all notions of space, must become one of its dimensions.

Space, moreover, is the form of extension. Everybody is extended. You can abstract, one by one, every attribute from a body except its quality of filling space. It is, indeed, possible to mistake space for an attribute of things, but certainly never time!

Bergson argues that, in the evolution of the idea of an extra dimension, motion is a necessary factor. Let us consider how the idea of a solid of three dimensions, is built up from the contemplation of plane figures. A square plane is supposed to be moved along in a direction perpendicular to its surface until a cube may be traced in the mind's eye; or a semi-circular plane is made to revolve about its diameter so as to trace a sphere. Thus motion, of which time is an implicit function, is required to create the idea of a body having a third, or extra, dimension. Arguing along these lines, Bergson lays down this general proposition: "that which is given as a motion in a continuum (space) of a given number of dimensions can be represented as a form in a continuum having one more dimension."

This is Bergson's apology for the introduction of time as a dimension of Minkowski's "world," because where you have motion, you must have the idea of time, and so an event, or happening, in a three-dimensional space becomes a world-point in a four-dimensional continuum.

In the case of the navigator, his time-recording chronometer is simply ticking off the units of distance along the latitudes on his chart from the index-meridian (Greenwich), which units are merely deduced from the differences between Greenwich time and local times. It is a solecism to speak of time as one of the dimensions in this connection; but then we confess that to speak of time as a dimension of space, seems to us always a solecism; though it may not be such to speak of a time-co-ordinate or a measure of time.

Nevertheless, it must be admitted that Kant, in describing time as the form of both internal and external sense, somewhat belies his axiom, "there is only one time." Moreover, in conjuring up a mental picture of time, he plots a sort of graph of time, contending, too, that "time has only one dimension:" this graph of Kant's is a straight line; upon it, sense-data are threaded, like beads, in the synthesis of apprehension, while, in the syntheses of reproduction and recognition, Kant seems to make them slip to and fro. The description of time as both the internal and external form of perception may have engendered the idea of two sorts of time, and the spatial quality of serial time may have arisen from the graph.

Einstein seems to have adopted Minkowski's space-time-continuum without any sort of ontological scrutiny of its elements; so it may assist in elucidating this puzzling aspect of Einstein's four-dimensional continuum if we consult Bergson.

In his TIME AND FREE WILL, Bergson writes:—"Time is the medium in which conscious states form discrete (disjunctive) series; this time is nothing but space and pure duration is something different."

From this, it would appear that he is contending that there are two sorts of time; duration, and serial time.

In his DURATION AND SIMULTANEITY, Bergson maintains that there are two sorts of time; psychical duration and mathematical (conventional) time. Besides, he argues that time cannot be measured until it be spatialised, and he says that spatialised time is not real time, not the time which is lived, but is a mere dimension of space.

Bergson denounces Minkowski's and, incidentally, Einstein's time as merely mathematical time, time which is measured.

We cannot help suspecting that Bergson means to denounce Einstein's time as merely a hollow measure of time; as a mathematical entity utilised for the calculation of any velocity in relation to the velocity of light: in fact, velocity (mathematically) is the differential co-efficient of space (distance) with respect to time.

By way of stripping Minkowski's time of all psychical significance, Bergson contends:—

"It is not the true time which is a memory of internal change, a melody which we hear with closed eyes, thinking only of its cadence in rhythm with the very flux of our inner life."

Einstein's time, the mathematical entity, coincides with true time, according to Bergson, only when the system of co-ordinates, or measuring standards, is at rest in relation to the system in which the events are occurring which are being studied. In every other case, mathematical time differs from psychical duration and the amount of the difference depends upon the relative velocity of the systems, there being a different time for each velocity.

Briefly, Bergson considers that there is only one true time, but several mathematical times, the distinction, generally speaking, lying between time as duration and time as series.

So that the series, which Kant describes as a succession of sense-data presented to the mind one after another in time, would be described by Bergson as a discrete (disjunctive) succession of sense-data spread out through a dimension of space which he calls mathematical time, or spatialised time which can be measured.

Moreover he writes: "what we call measuring time is nothing but counting simultaneities and the connecting link between space and duration (Bergson's real time) is simultaneity which might be described as the intersection of time and space."

Now this appears to be a somewhat clumsy explanation, because, if it can cut arcoss space, duration too must be spatialised, and be a mere mathematical entity.

But we shall endeavour to show that serial time cannot always (in relation to causality, for instance) be denounced as not real time, but a mere dimension of space.

Now, Kant defines causality thus. "Causality involves the idea of ordered succession of phenomena in time, cause being always antecedent and effect consequent, and the terms in the series of phenomena presented in succession must be so related that, without the former, the latter cannot be."

So here we have ordered succession of phenomena in time, that is to say, Bergson's discrete series; furthermore, all idea of simultaneity must be emphatically foreign to phenomena which are essentially antecedent and consequent.

Now, nothing could possibly give a more definite notion of true time—time in its chronological sense, independent of space—than causality and yet nothing could give a more distinct idea of serial time.

Indeed, if time, in this connection, were the mere mathematical time, such as Bergson attributes to the proposition of Einstein's General Theory of Relativity, then, by carefully choosing the relative velocity of the observer and the system in which given events are occurring, an effect might be made antecedent to its cause, which is absurd!

Thus, between Kant and Bergson, Einstein is on the horns of a dilemma: his time cannot be either the serial time which is the essential factor of causality, or Bergson's psychical duration; yet time, as a fourth dimension of his continuum, is necessary for the General Theory, because his "intervals," or geodesic lines, through curved space in the space-time-continuum, are dependent not only upon space, but on velocity and so, implicitly, on time: as Bergson would express it: the motion in three dimensions becomes the form in four.

It seems to us that the only escape from this dilemma is by regarding Einstein's time as bearing the same sort of relation to true time as velocity bears to motion: the one is the quantity, or degree, of the other's flux: a time-co-ordinate (or measure) is not so repugnant an idea as a time-dimension.

It must not be assumed, however, because the velocity of light is Einstein's standard for measuring all velocities, that, within the universe, it is an absolute quantity. On the contrary, it varies inversely with the gravitational intensity: thus the value of C is approximately 300,000 kilometres a second to an observer upon the Earth, but it is different to observers on other planets which are in zones of more or less intensity.

It follows that a ray of light, traversing the sun's gravitational field, from one remote domain to another, through the intensest zone, does not travel with uniform velocity, but it is subject alternatively to retardation and to acceleration. If a graph of its journey were traced on a chart with co-ordinates enabling the varying velocities to be plotted, a curved path would be recorded, thus harmonising with the idea that the geodesic line pursued by a light-ray in curved space must be a curve.

The velocity of light varying inversely with the intensity of the gravitational field, "an atom"—in the language of Einstein—"emits light of a frequency which depends on the potential of the gravitational field in which it is situated."

Now the effect of this is shown by a shifting of the spectral lines.

Thus the spectral lines of a ray emanating from a glowing gas on the sun will be shifted nearer to the red end of the spectrum than the corresponding spectral lines of the corresponding spectrum of the corresponding gas on the Earth, because the gravitational intensity is greater at the sun than at the Earth.

Here, then, is a method for determining the potential of any gravitational field in the universe, Einstein having given in his book a formula for calculating the amount of displacement of the spectral lines due to this cause.

Professor Eddington, using Einstein's formula, has made some astonishing discoveries relating to the Companion of the great star Sirius and his calculations have been confirmed by the observations of Mr. Adams.

These discoveries constitute the third of the triumphant verifications of the General Theory of Relativity.

If the time-dimension in Minkowski's "world" be not true time, as Bergson alleges, it appears, nevertheless, to have furnished Einstein with a mathematical device which has enabled him to solve the great astronomical problems with more precision than the mighty Newton.

VII.

CAN EINSTEIN'S PROPOSITIONS BE JUSTIFIED METAPHYSICALLY?

NOTWITHSTANDING the most adroit casuistry, Kant and Einstein differ widely. The space and time-dimensions of Minkowski's "world" are not the pure intuitions of the great transcendental philosopher. How can they be? How can the knowledge of any fourth dimension exist *a priori* in the human mind? What, then, is Einstein discussing?

Are the objects presented to his mind, in the four-dimensional continuum, phenomena, or is his reason speculatively forming transcendent ideas of things in themselves?

In his arguments relating to the constancy of light in the Special Theory, he seems to be dealing with the appearance of things, so that his reason is confining its activities to objects of possible experience.

But, when he writes: "from a happening in three-dimensional space, physics becomes an existence in the four-dimensional 'world,'" he clearly intends to express his belief that the "world" is a sort of spatial ocean in which things in themselves exist and extend themselves, it is not Kant's pure form of perception through which impressions take shape as phenomena.

Thus to Einstein physics are the laws of things in themselves, not the laws of the mind which, in the Kantian system, co-ordinate and arrange the manifold of phenomena (which is Nature) prescribing their behaviour.

And these points of view are only reconcilable if the laws of the mind be regarded as evolving towards absolute harmony with the laws of things: as human knowledge increases, despite occasional reactions, despite the ebbing and flowing of the tides of human wisdom, so each theory based on fresh discoveries may be approaching coincidence with the laws of things in themselves.

Kant says: "although we can never know things in themselves, we can infer with certainty that they exist," they are at the back of all phenomena.

If that be so, we can infer that there are laws too of things in themselves; and the laws of physics, the laws of the Science of Nature, such as Kant defines them, may be in process of evolution: thus the theory of aether is a law of the mind applied to phenomena, as are likewise the theories of Newton and Einstein, and each in its way, may be a stride nearer the laws of things than—say—the Ptolemaic theory: every new theory approaching closer to "those mysteries of which philosophism has not yet dreamt." Nevertheless the laws of the mind may never attain their end; they may be asymptotic, for ever approaching, yet destined never to reach their goal.

Kant has explained that illusions arise from the fallacy of reasoning about phenomena having assumed them to be things in themselves. We are tempted—being stalwart in the Kantian faith—to denounce all four-dimensional mathematics as dialectical, as appertaining to the logic of illusions. Within the domain of pure mathematics, these Gaussian equations as used by Einstein are valid, but the graphs of them (which can never be drawn) must be chimeras, and the curved space may be a figment. Although a figment, it is, nevertheless, a hypothesis[1] which might be justified by a pragmatist, because, upon it, a mathematical system perfectly logical at every step, save possibly its premises, has been built which has proved the most efficient device for the solution of astronomical problems.

Einstein believes that the whole of the universe curves into itself, so that, though boundless, the universe is not infinite, but Kant says, "we can never know the whole of the universe, because we can never know the totality of all phenomena, for neither in regression nor in progression can

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[1] A hypothesis, or what Einstein himself calls "the fundamental assumptions, or the so-called axioms."

our minds ever reach the beginning or the end of the stream of phenomena which are threaded on time, the pure intuition of our perception." Just as the clown in the circus, tripping over the carpet, which he is carrying like an apron, can never gather up the end of it.

It follows from this argument of Kant's that eternity is an illusory idea which results from time being a condition, or limitation, of human intuition; similarly, infinity is an illusory idea arising from three-dimensional space being a condition, or limitation, of human intuition.

Thus, of eternity and infinity, it may be said that, in the domain of ultimate reality, there are no such things; we only speak of them by reason of the circumscription of the human faculties.

Einstein's Theories force us by a different path to the conviction that the continuum in which the universe has its being and wherein the heavenly bodies extend themselves, is only infinite to our three-dimensional minds, just as the surface of the Earth on which we live might appear infinite to a two-dimensional mind.

Similarly the restrictions of our faculties reduce our knowledge of time and motion to unsatisfactory limitations.

Einstein has taught us to infer that the continuum, in which the world as we know it exists, has something more than three-dimensions, that there is some relation between time and space which our minds are inadequate to apprehend and that the change of things and the flux of events proceed in some manner of which motion and time are merely faint notions.

Einstein, as we have remarked, is not a metaphysician, indeed he has disavowed any metaphysical implication in advancing his Theories; he has approached these vast problems from the point of view of a physicist and a mathematician, and he is a very great physicist and a very great mathematician.

Einstein has not floundered in the metaphysical morass which over-whelmed the Cartesians and, after them Leibnitz, Berkeley and Hume, and he has transcended the quagmire of the illusions so magnificently expounded by Kant in his Transcendental dialectic, because he recks little for the morasses which engulf dialecticians; he is not one to go astray by method.[1]

There is often a world of difference between the Pure Mathematician and the Metaphysician. The proofs of pure mathematics, within their own domain, are so convincing that we are hypnotized by them; but we must never lose sight of their limitations when metaphysically applied. Their failure to so recognise these limitations wrecked the Cartesians.

The thinker, who refuted Des Cartes's arguments on the absolute reciprocity of relative motion, by contending that Peter, lolling in his garden-chair, could never be in doubt as to which of the two was in motion, himself or the panting, perspiring Paul running past him, composed a fable which all mathematicians should study.

[1] Metaphysics, according to a French wit, is *l'art de s'égarer avec méthode.*

VIII.

FINAL REMARKS.

In the "Prolegomena," Kant writes, "Physics can never rival Pure Mathematics," meaning that the proofs in the Science of Nature can never impress us with the apodictical certainty which is the peculiar quality of mathematical judgments, because mathematics rests upon its own evidence, whereas Physics rests upon experience and its thorough confirmation.

It necessarily follows that the laws of Physics are being continually modified by fresh discoveries and by improvements in the instruments which are used by physicists.

If, for instance, more delicate instruments were able to reveal the variability of the velocity of a light-ray according to the relative movements of the observer with respect to the body emitting the light, then the Special, or Restricted, Theory of Relativity would cease to be valid.[1]

Inasmuch as Physics rests upon experience and its thorough confirmation, there must always arise conflicts of opinion between the greatest doctors of science.

To Kant, the form of all thought was ideal, its matter was real; and the sensuous impressions were the material of all experience. It is doubtful whether Kant could remain of that opinion if he were alive to-day, because physical research has revealed the existence of many things which cannot be apprehended through our senses; the ultra-violet rays, for instance.

Einstein begins by denying the existence of aether, although space, as he conceives it, is endowed with physical qualities and he admits that it does not seem possible for empty space to have the physical qualities which are necessary to his General Theory of Relativity, nor for light to be propagated through empty space.

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[1] But not the General Theory.

Nevertheless, he appears to believe that, remote from gravitational spheres, space is a vacuum, not a plenum.

Einstein contends that force cannot act across empty space, and that nothing can leap instantaneously across space but everything must move with some defined velocity, like a ray of light which traverses space with the velocity to which we have so often alluded and which is constant for all observers under all conditions, in the same intensity of the same gravitational field.

Einstein has furnished three very convincing proofs of the truth of his theories: the curvature of light-rays passing through the curved space of the sun's gravitational field, the rotation of Mercury's orbit and the measurement of the potential of a gravitational field by the amount of displacement of the spectral lines.

On the other hand, there is not a scintilla of evidence of an empirical nature that aether exists. It has defied all tests undertaken to detect its presence.

To most scientists, indeed, aether is no more than a convenient hypothesis for explaining the peculiar qualities and functions of electrical waves, etc., nevertheless, Sir Oliver Lodge maintains that it must exist. The universe, as he knows it, could not be without it; without aether there could be no science of physics!

Indeed, in his latest work, Sir Oliver Lodge reverts to the metaphysical standpoint of Spinoza. His aether is Spinoza's Substance with its attributes of Thought and Extension; it is the link between mind and matter; it is the god of the Cartesians.

Thus Sir Oliver Lodge is reduced to the same expedient for proving the existence of aether, as Kant in solving the metaphysical problems.

Kant, let it be remembered, having exhausted all the arts of dialectic in a vain effort to solve the great metaphysical problems, boldly declared that, without Free Will, Immortality and God, there could be no moral law; and the categorical imperative is there, in the soul of every man, to prove that there is a moral law.

Thus, belief in the existence of aether may he held with all the strength of the rock on which all faith is built, but in the present stage of our knowledge it cannot be proved empirically; so any endeavour to prove its presence must come under that faculty of logic which Kant calls the Transcendental Dialectic, and, the Dialectic is the logic of illusions!

Einstein may have to suffer a reproach for building his system upon mollusks, but his rivals build theirs, too, upon illusions!

Whatever be the fate of Einstein's theories, the thinking world can never return to the classical physics of Newton—a science based upon the mathematics of Euclid, inapplicable in a world of shifting masses in a constant state of flux and change.

Einstein has brought about a revolution in thought. It may not have the same far-reaching consequences as those which sprang from the discoveries of Galiléo; it may neither shatter faiths nor dethrone pontiffs, for his theories must remain, for ever, caviare to the general.

APPENDIX.

A few Lorentz-Einstein equations of transformation.

The velocity of A with respect to an observer B $= v$.
The velocity with which D is projected from A $= w$.
The velocity of light $= c$.

Then the velocity of D with respect to B $= \dfrac{v + w}{1 + \dfrac{vw}{c^2}}$

The length of a metre rod l moving with a velocity v with respect to an observer at rest $= l\sqrt{1 - \dfrac{v^2}{c^2}}$

A clock which ticks once in a second and which is moving with a velocity v with respect to an observer at rest.

To the observer, the interval between two ticks will not be one second, but it will be $\dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ seconds.

The fourth Lorentz equation of transformation to which reference has been made: Suppose x^1, y^1, z^1, t^1 to be the four co-ordinates of a system K^1 moving parallel to x_1 the axis of another system K_1 with relative velocity v, then

$$x^1 = \frac{x - vt}{\sqrt{1 - \dfrac{v^2}{c^2}}};\ y^1 = y;\ z_1 = z, \text{ and } t^1 = \frac{t - \dfrac{v}{c^2}x}{\sqrt{1 - \dfrac{v^2}{c^2}}}.$$

If a body take up an amount of energy E, then its inertial mass increases by an amount $\dfrac{E}{c^2}$.

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